GEOMETRY.

489. Proposed by NATHAN ALTSHILLER, The University of Colorado.

The parallels to the asymptotes a, b of a given hyperbola, drawn from a variable point of the curve, meet a and b in P, Q respectively. The line PQ envelops an hyperbola whose asymptotes are a and b.

490. Proposed by ELMER E. MOOTS, University of Arizona.

In any quadrilateral $ABCD$, let AC and BD be the diagonals intersecting in K. On AC, lay off CR equal to AK. Join B and R. Connect the middle point G of BR with D. On GD, lay off GM equal to $\frac{1}{3}GD$. Show that M is the center of gravity of the quadrilateral.

491. Proposed by N. P. PANDYA, Sojitra, India.

In a triangle $mx = b$ and $nx = c$, determine a relation between m, n, x, A and s and solve it for x.

CALCULUS.

407. Proposed by PAUL CAPRON, Annapolis, Maryland.

A coffee pot in the form of a conical frustum, 10 inches high, with a lower base 8 inches in diameter and an upper base 6 inches in diameter, is held on a slant so that the lower base is barely covered by the coffee within, and the upper base is barely uncovered. How much coffee does the pot contain?

408. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

The ellipse $(x^2/81) + (y^2/16) = 1$ is revolved around the y-axis. Find the area of the surface generated.

409. Proposed by B. J. BROWN, Victor, Colorado.

Integrate the equation

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{1}{x+y}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) - \frac{2}{(x+y)^2} z = 0.$$

MECHANICS.

326. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A uniform beam, of length $2l$, rests in equilibrium against a smooth vertical wall and upon a peg at a distance a from the wall; show that the inclination of the beam to the vertical is

$$\sin^{-1}\left(\frac{a}{l}\right)^{\frac{1}{3}}.$$

327. Proposed by C. N. SCHMALL, New York City.

An inclined plane makes an angle ϕ with the horizontal plane, and from its foot a body is projected upward at an angle ψ to the plane, and with velocity v. Show that it will strike the plane *perpendicularly* if $\tan \psi = \frac{1}{2} \cot \phi$ and that its range up the plane in that case will be

$$\frac{2v^2 \sin \phi}{g(1 + 3\sin^2 \phi)}.$$

NUMBER THEORY.

244. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Determine the rational value of x that will render $x^2 + px + q$ a perfect square. What value of x will render $x^2 - 7x + 2$ a perfect square?

245. Proposed by NORMAN ANNING, Chilliwack, B. C.

When all the letters denote positive integers and when the a's are primes of the form $4k + 1$, the equation

$$x^2 + y^2 = (a_1a_2a_3\cdots a_m)^n,$$

has, in Legendre's notation, $E((n+1)^m/2)$ solutions. Show that in 2^{m-1} of these solutions x and y are relatively prime.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

445. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{cn}{a} - \binom{a}{1}\binom{cn-c}{a} + \binom{a}{2}\binom{cn-2c}{a} - \cdots + (-1)^a\binom{cn-ca}{a}.$$

I. SOLUTION BY THE PROPOSER.

This is a generalization of the series forming the first member of equation (2) in the Proposer's solution of problem No. 424 (*June* 1915 issue, p. 205), obtained by multiplying the upper indices $n, n-1, n-2, \cdots$ by the constant factor c. Following the method employed in that solution, we find that the given series equals

$$\Delta^a\binom{cn-ca}{a},$$

the finite differences being taken with respect to n.

Now

$$\Delta_x\binom{cx}{a} = \binom{\overline{cx+1}}{a} - \binom{cx}{a},$$

the second member of which may be written in the following form:

$$\binom{cx+c}{a} - \binom{cx+c-1}{a} + \binom{cx+c-1}{a} - \binom{cx+c-2}{a} + \binom{cx+c-2}{a} - \cdots$$
$$+ \binom{cx+1}{a} - \binom{cx}{a},$$

since all these terms, except the first and last, alternately cancel each other. Combining the terms in pairs and noticing that

$$\binom{cx+c}{a} - \binom{cx+c-1}{a} = \binom{cx+c-1}{a-1}, \quad \binom{cx+c-1}{a} - \binom{cx+c-2}{a} = \binom{cx+c-2}{a-1},$$

etc., we have

$$\Delta_x\binom{cx}{a} = \binom{cx+c-1}{a-1} + \binom{cx+c-2}{a-1} + \cdots + \binom{cx+1}{a-1} + \binom{cx}{a-1},$$

which means that the first difference, taken with respect to x, of the binomial coefficient $\binom{cx}{a}$ having for its lower index a, equals the sum of c binomial coefficients, each having for its lower index $a-1$.

In the same manner, the second difference $\Delta_x^2\binom{cx}{a}$ may be expressed as the sum of $c\cdot c = c^2$ binomial coefficients, each having $a-2$ for its lower index; and continuing this process, we find that the ath difference $\Delta_x^a\binom{cx}{a}$ may be expressed as the sum of c^a binomial coefficients, each having for its lower index $a-a=0$ and hence each equal to 1. In other words,

$$\Delta_x^a\binom{cx}{a} = c^a,$$

and, similarly,

$$\Delta_n^a\binom{\overline{cn-a}}{a} = c^a.$$